Karlsruhe Institute of Technology

A Theory of C-Style Memory Allocation

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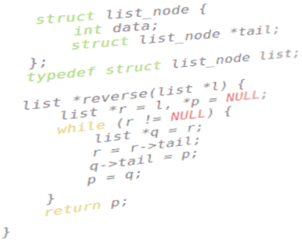
Research Group “Verification meets Algorithm Engineering”



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- Solution:

An SMT theory for reasoning about memory access correctness

- Precise formalization of `malloc` and `free`
- Opportunities for simplification of formulas using rewrite rules
- Modular and local reasoning

LLBMC — Low-Level Bounded Model Checking

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  - Low-Level: Embedded devices, systems code, ...
  - Software: Programs written in C/C++
  - Bounded: restricted number of nested function calls and loop iterations
  - Model Checking: bit-precise static analysis

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  - Bounded: restricted number of nested function calls and loop iterations
  - Model Checking: bit-precise static analysis
- Properties checked:
  - Built-in properties: invalid memory access, use-after-free, double free, arithmetic overflow, division by zero, . . .
  - User-supplied properties: `assert` statements

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- Bugs manifest themselves in finite runs of the program
- Software bounded model checking:
  - Analyze only bounded program runs
    - Restrict number of nested function calls and inline functions
    - Restrict number of loop iterations and unroll loops
  - Property checking becomes decidable using an SMT solver

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  - Closer to the program that is actually run on the machine
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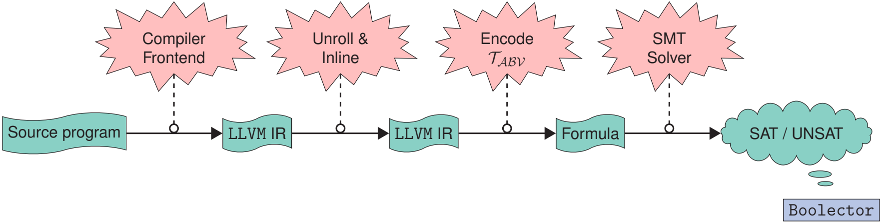

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%x = load i8* %p
store i8* %p, i8 %x
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$$x = \mathit{read}(a, p)$$
$$\widehat{a} = \mathit{write}(a, p, x)$$

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$$\begin{aligned} \widehat{a} &= \mathit{write}(a, p, x) \\ x &= \mathit{read}(\widehat{a}, p) \end{aligned}$$

Memory Related Program Bugs

- Kinds of memory related bugs:
  - `load` from a non-allocated region of memory
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- Need to be handled atop of $\mathcal{T}_{\mathcal{ABV}}$

Encoding Memory Access Correctness in SMT

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- Current approach in `LLBMC` (SSV '10):
  - Introduce explicit heap states: $h, h', \ldots$
  - Memory access correctness constraints explicitly encoded in $\mathcal{T}_{\mathcal{ABV}}$
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$$\texttt{accessible}(h, q, t) \quad \equiv \bigvee_{\substack{h' \preceq h \\ I:\, h' = \texttt{malloc}(\widehat{h'}, p, s)}} c_{\texttt{exec}}(I) \;\wedge\; \neg\texttt{deallocated}(h', h, p) \;\wedge\; \texttt{contained}(p, s, q, t)$$

$$\texttt{deallocated}(h, h', p) \quad \equiv \bigvee_{\substack{h \preceq h^* \preceq h' \\ I:\, h^* = \texttt{free}(\widehat{h^*}, q)}} c_{\texttt{exec}}(I) \;\wedge\; p = q$$

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- Goals of this work:
  - Precise formalization of `malloc` and `free` as an SMT theory
  - Opportunities for simplification of formulas using rewrite rules
  - Modular and local reasoning: partial heap state history suffices

Motivation: McCarthy's Theory of Arrays

McCarthy's *read*-over-*write* axioms:

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\begin{aligned}
i = j \quad &\Rightarrow \quad read(write(a, i, x), j) = x \\
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Reduction to equality logic:

1. Apply axioms in the form of the rewrite rule

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More efficient approaches exist, e.g., `Boolector`'s lemmas-on-demand

Introducing $\mathcal{T}_{\mathcal{H}}$

- Objects of type H encode information about the heap state
  - Contain the whole history of `malloc` and `free` operations
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Functions of type H:

- ε — Heap without any allocation
- $\mathtt{malloc}(h, p, s)$ — Allocate region $[p, p + s)$ (if possible)
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Auxilliary functions:

$\texttt{mallocsize}(h, p)$	Size of the memory region starting at p (if currently allocated)

Predicates of $\mathcal{T}_{\mathcal{H}}$

Predicates:

- $\texttt{accessible}(h, p, s)$ Is $[p, p + s)$ contained in an allocated region?
- $\texttt{freeable}(h, p)$ Is p the start of an allocated region?
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The semantics of the predicates is specified using axioms

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$\texttt{mallocable}(h, p, s)$ Can $[p, p + s)$ be allocated?

- $[p, p + s)$ can be allocated if it does not overlap with any allocated memory region
- Various possible axiomatizations:
  - Stack-like: p needs to be “on the right” of all regions that have been allocated before
  - Axiomatize non-overlap precisely
  - …

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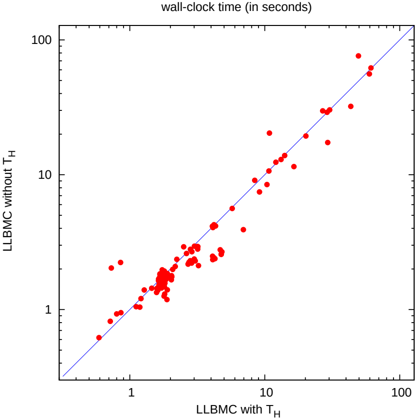
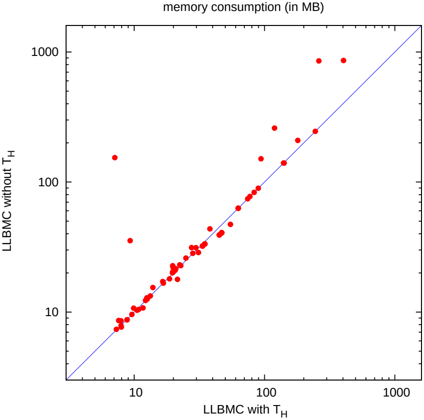
Evaluation

Comparison of the current approach (SSV '10) and $\mathcal{T}_{\mathcal{H}}$ in LLBMC on 97 C-programs





Details

	#mallocs / #frees	#accessible	time		memory	
			SSV '10	$\mathcal{T}_{\mathcal{H}}$	SSV '10	$\mathcal{T}_{\mathcal{H}}$
sparsemem	129 / 51	8374	76.2	49.5	861	404
binary-tree	127 / 127	3048	7.5	9.1	150	94
inplace-reverse	100 / 100	1800	20.4	10.8	260	119
wcet-bsort100	3 / 0	120204	12.4	12.1	246	246
wcet-statemate	106 / 0	2816	2.2	0.9	35	9

Conclusions and Future Work

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- Potential performance increases:
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- Open Questions:

1. Can $\mathcal{T}_{\mathcal{H}}$ be integrated into SMT solvers?
2. Is the lemmas-on-demand approach applicable?